Cooperative Underwater Navigation

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JNRR, Vittel
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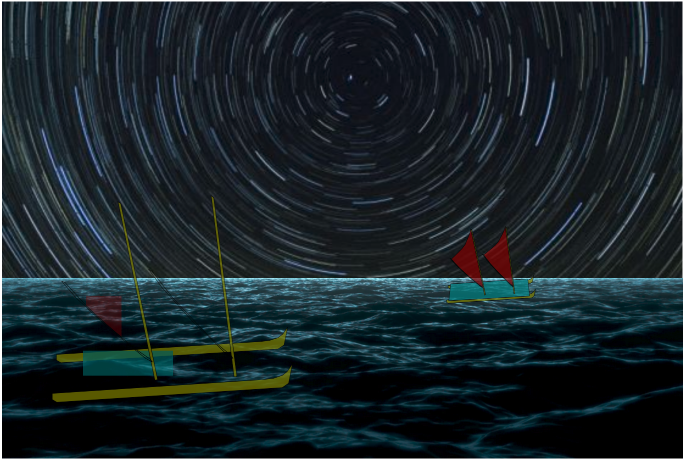
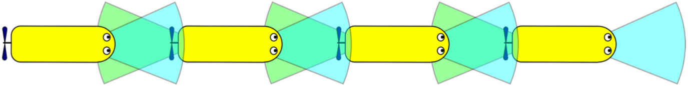
Cooperative robotics?

Polynesian navigation

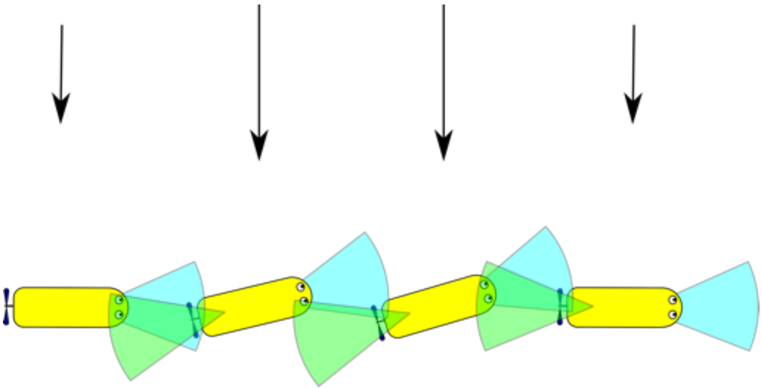


Find the route without GPS, compass and clocks with *wa'a kaulua*[2]

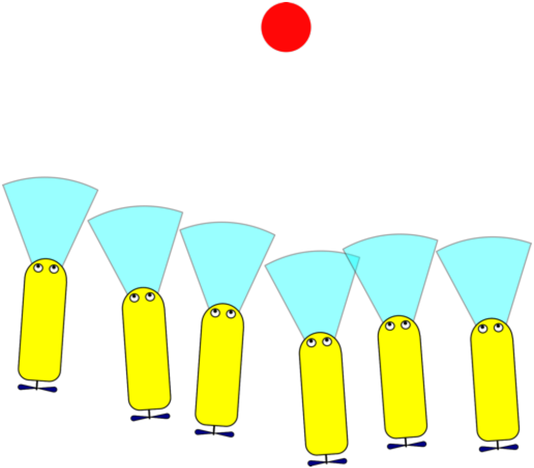
Alignment to keep the heading in case of clouds



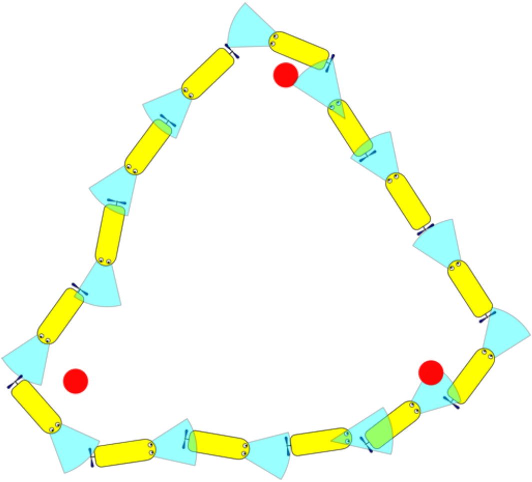
More inertia, more predictable



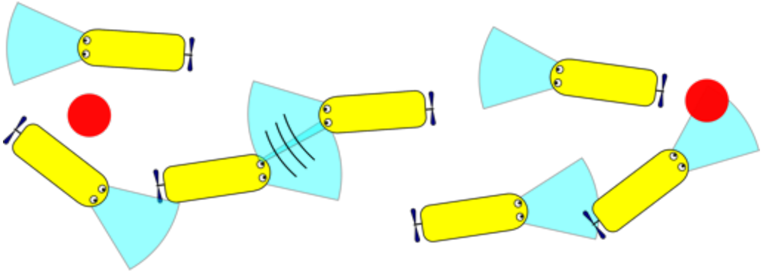
Internal deformations provide information



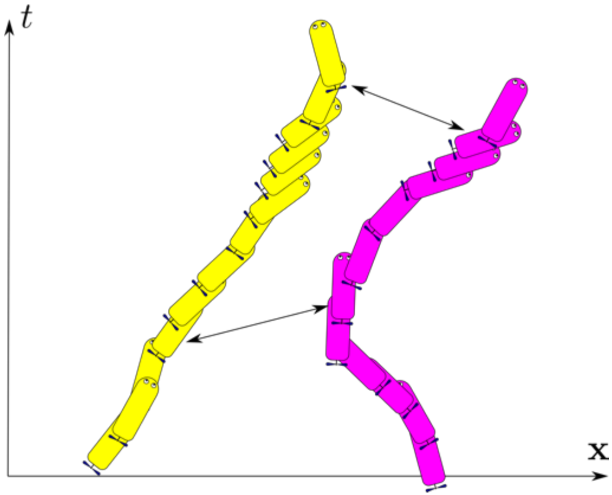
Explore further



Virtual chain: localization ↔ proprioception



With communication we can do more



Perception of others rigidifies the evolution of the group

Secure a zone

Secure a zone

INFO OBS. Un sous-marin nucléaire russe repéré dans le Golfe de Gascogne

Le navire a été repéré en janvier. Ce serait la première fois depuis la fin de la Guerre Froide qu'un tel sous-marin, doté de missiles nucléaires, se serait aventuré dans cette zone au large des côtes françaises.

Bay of Biscay 220 000 km^2

An intruder

- Several robots $\mathscr{R}_1, \dots, \mathscr{R}_n$ at positions $\mathbf{a}_1, \dots, \mathbf{a}_n$ are moving in the ocean.
- If the intruder is in the visibility zone of one robot, it is detected.[4]

Complementary approach

- We assume that a virtual intruder exists inside $\mathbb{G}$.
- We localize it with a set-membership observer inside $\mathbb{X}(t)$.
- The secure zone corresponds to the complementary of $\mathbb{X}(t)$.

Assumptions

- The intruder satisfies

$$\dot{\mathbf{x}} \in \mathbb{F}(\mathbf{x}(t)).$$

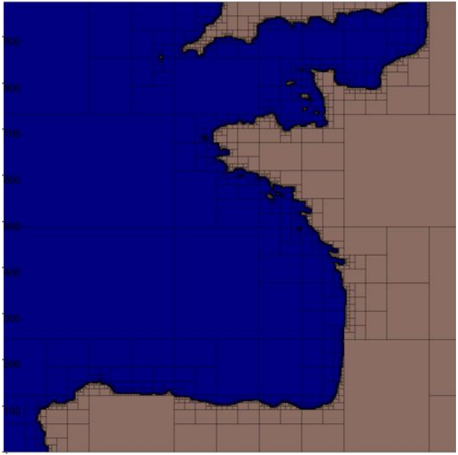
- Each robot $\mathscr{R}_i$ has the visibility zone $g_{\mathbf{a}_i}^{-1}([0, d_i])$ where d_i is the scope.

Theorem. An (undetected) intruder has a state vector $\mathbf{x}(t)$ inside the set

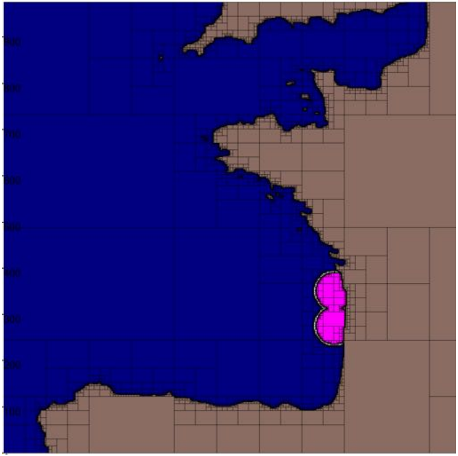
$$\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt)) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty]),$$

where $\mathbb{X}(0) = \mathbb{G}$. The secure zone is

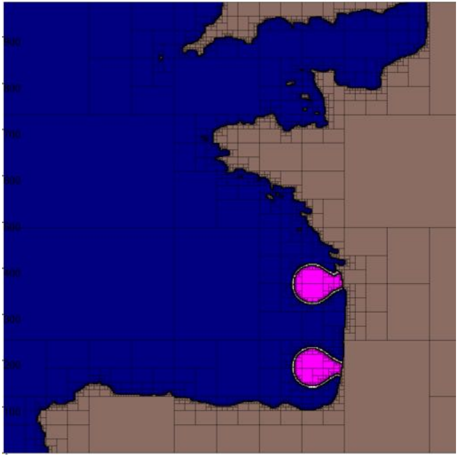
$$\mathbb{S}(t) = \overline{\mathbb{X}(t)}.$$



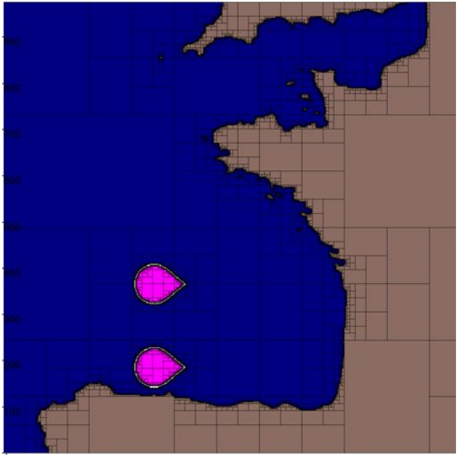
Set $\mathbb{G}$ in blue

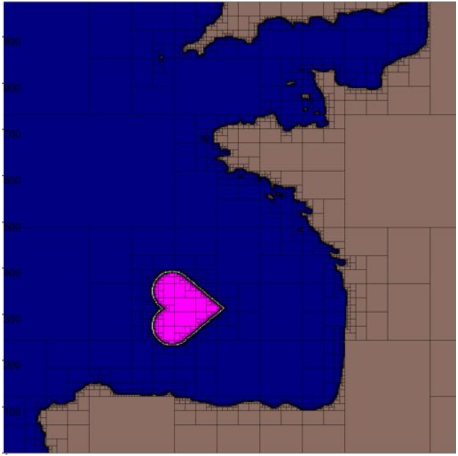


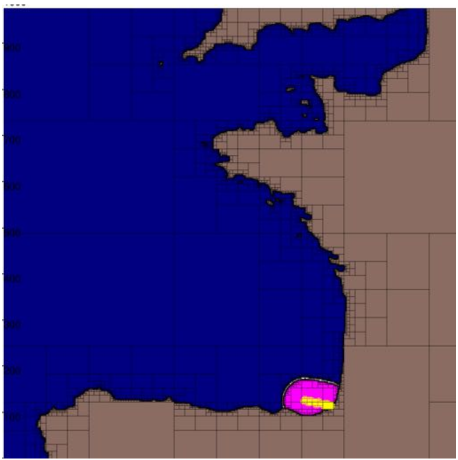
Magenta: $\mathbb{G}\cap\bigcup_i g^{-1}_{\mathbf{a}_i(t)}([0,d_i(t)])$ Blue:$\mathbb{G}\cap\bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t),\infty])$

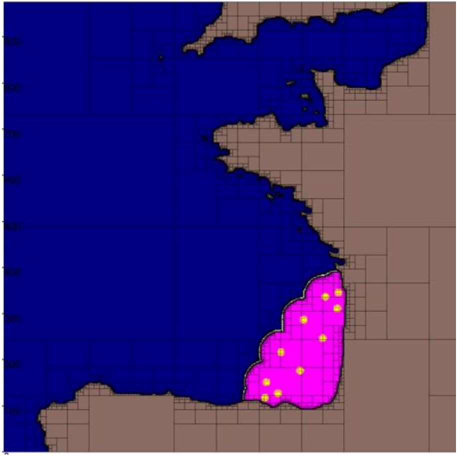


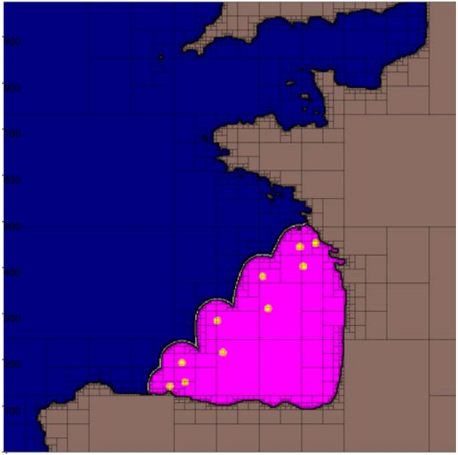
Blue:
$$\mathbb{X}(t) = \mathbb{G} \cap (\mathbb{X}(t-dt) + dt \cdot \mathbb{F}(\mathbb{X}(t-dt))) \cap \bigcap_i g^{-1}_{\mathbf{a}_i(t)}([d_i(t), \infty]).$$

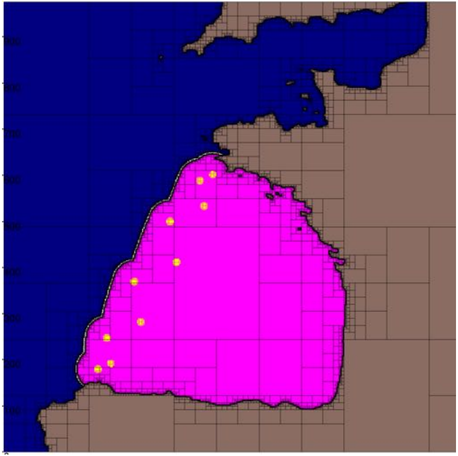






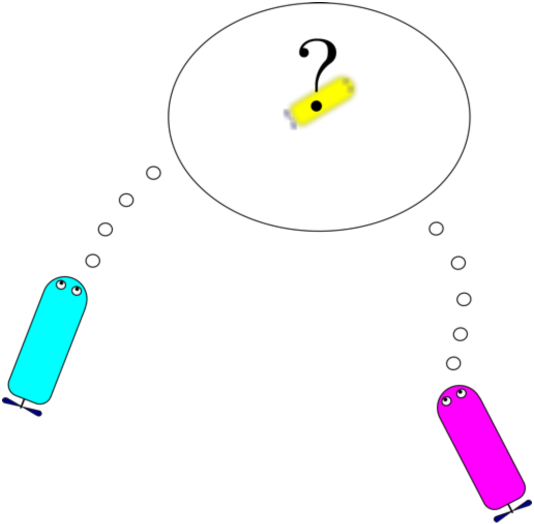


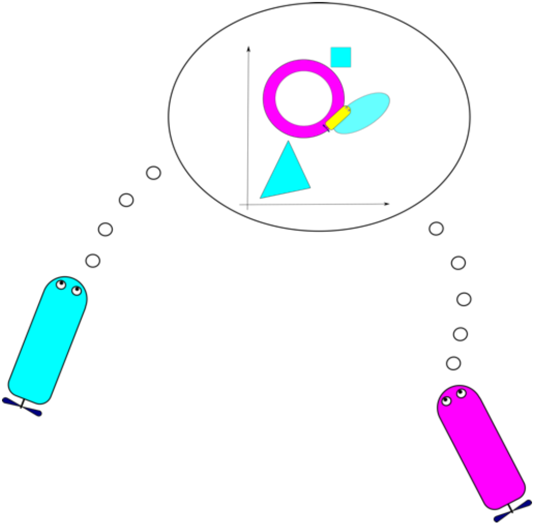


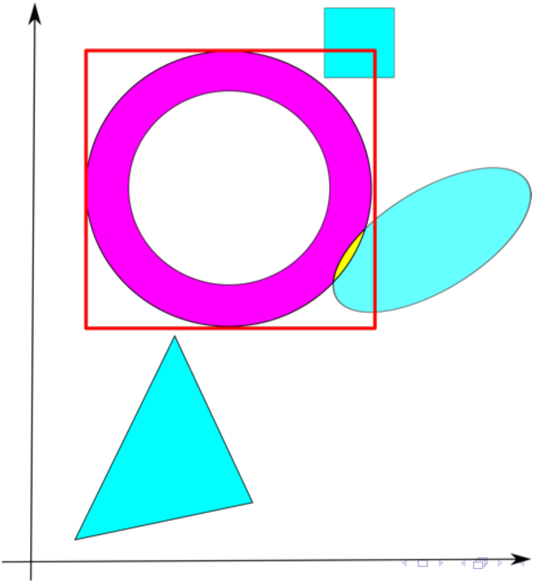


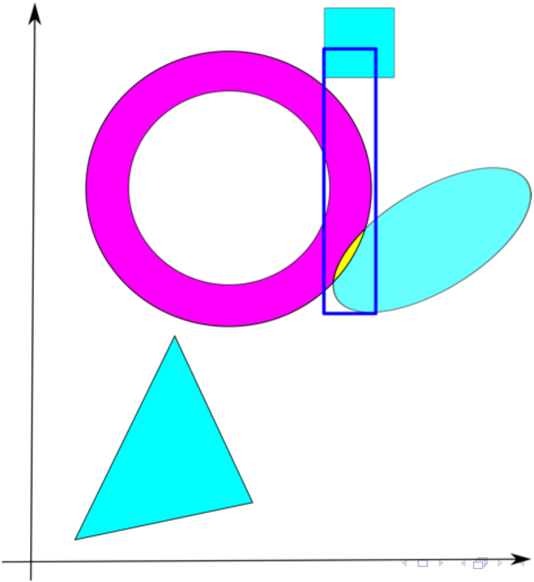
Cooperative solving

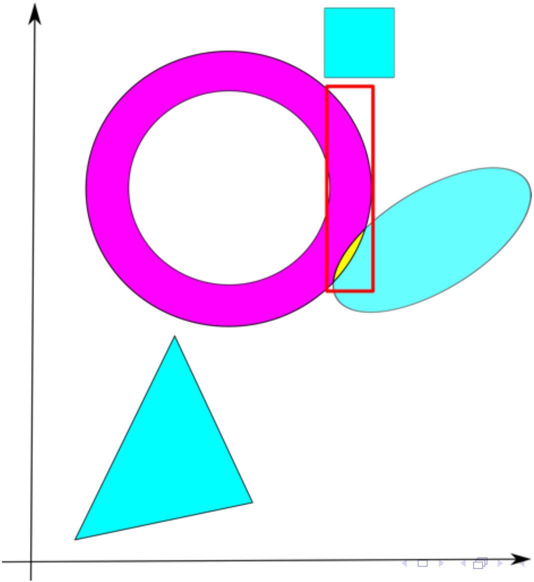
Cooperative solving

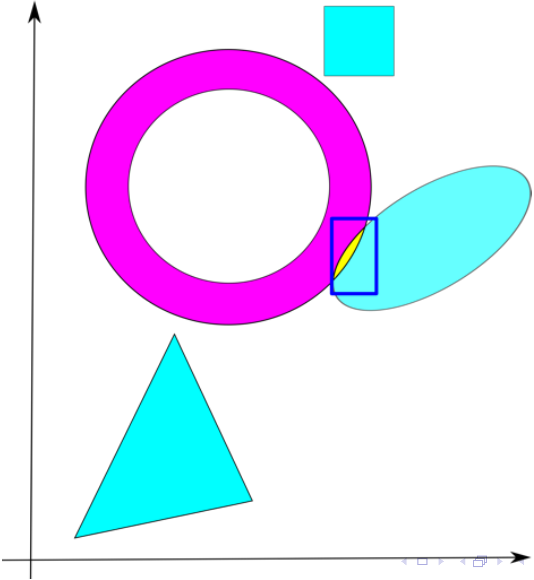








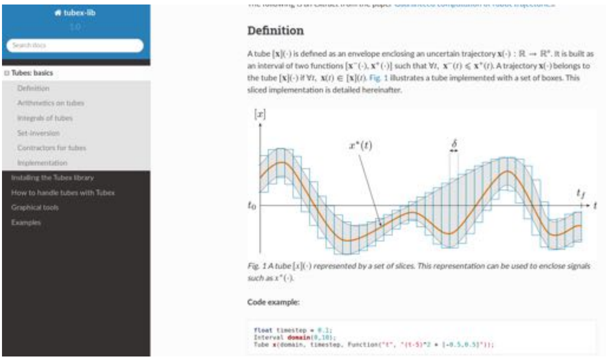




Youtube

With many outliers

Cooperative localization



http://www.simon-rohou.fr/research/tubex-lib/ [3]

Cooperative localization

Classical state estimation

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t),\mathbf{u}(t)) & t \in \mathbb{R} \\ \mathbf{0} & = & \mathbf{g}(\mathbf{x}(t),t) & t \in \mathbb{T} \subset \mathbb{R}. \end{cases}$$

Space constraint $\mathbf{g}(\mathbf{x}(t),t) = 0$.

Example.

$$
\begin{cases}
\dot{x}_1 = x_3 \cos x_4 \\
\dot{x}_2 = x_3 \cos x_4 \\
\dot{x}_3 = u_1 \\
\dot{x}_4 = u_2 \\
(x_1(5) - 1)^2 + (x_2(5) - 2)^2 - 4 = 0 \\
(x_1(7) - 1)^2 + (x_2(7) - 2)^2 - 9 = 0
\end{cases}
$$

With time-space constraints

$$\begin{cases} \dot{\mathbf{x}}(t) & = & \mathbf{f}(\mathbf{x}(t),\mathbf{u}(t)) & t \in \mathbb{R} \\ \mathbf{0} & = & \mathbf{g}(\mathbf{x}(t),\mathbf{x}(t'),t,t') & (t,t') \in \mathbb{T} \subset \mathbb{R} \times \mathbb{R}. \end{cases}$$

Example. An ultrasonic underwater robot with state

$$\mathbf{x} = (x_1, x_2, \dots) = (x, y, \theta, v, \dots)$$

At time t the robot emits an omni-directional sound. At time t' it receives it

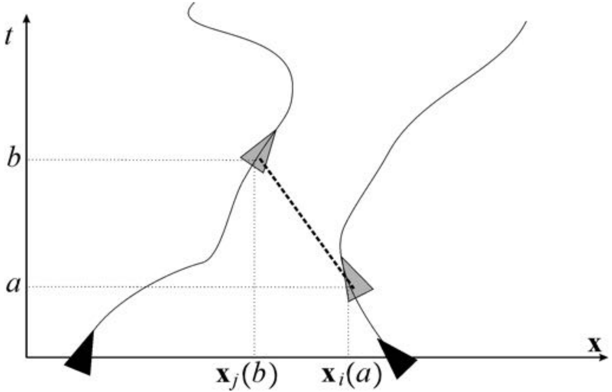
$$\left(x_1 - x_1^{'}\right)^2 + \left(x_2 - x_2^{'}\right)^2 - c\left(t - t'\right)^2 = 0.$$

Consider n robots $\mathscr{R}_1, \dots, \mathscr{R}_n$ described by

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i), \mathbf{u}_i \in [\mathbf{u}_i].$$

Omnidirectional sounds are emitted and received.
A *ping* is a 4-uple (a, b, i, j) where

- a is the emission time,
- b is the reception time,
- i is the emitting robot
- and j the receiver.


t
b
a
x
$\mathbf{x}_j(b)$
$\mathbf{x}_i(a)$

With the time space constraint

$$\begin{array}{l} \dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i), \mathbf{u}_i \in [\mathbf{u}_i]. \\ g\left(\mathbf{x}_{i(k)}(a(k)), \mathbf{x}_{j(k)}(b(k)), a(k), b(k)\right) = 0 \end{array}$$

where

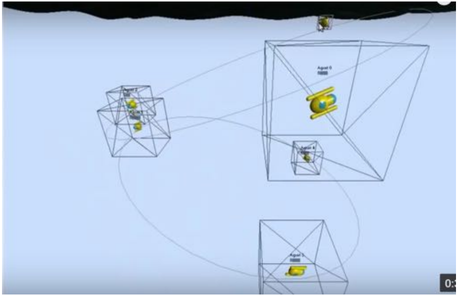
$$g(\mathbf{x}_i, \mathbf{x}_j, a, b) = \|x_1 - x_2\| - c(b - a).$$

Clocks are uncertain. We only have measurements $\tilde{a}(k), \tilde{b}(k)$ of $a(k), b(k)$ thanks to clocks h_i. Thus

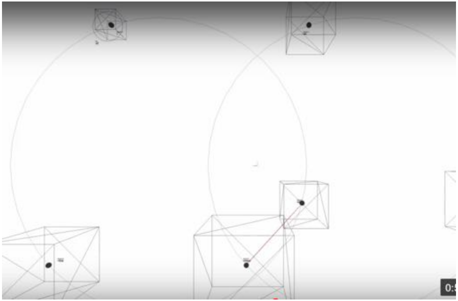
$$
\begin{array}{l}
\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i), \mathbf{u}_i \in [\mathbf{u}_i]. \\
g\left(\mathbf{x}_{i(k)}(a(k)), \mathbf{x}_{j(k)}(b(k)), a(k), b(k)\right) = 0 \\
\tilde{a}(k) = h_{i(k)}(a(k)) \\
\tilde{b}(k) = h_{j(k)}(b(k))
\end{array}
$$

The drift of the clocks is bounded

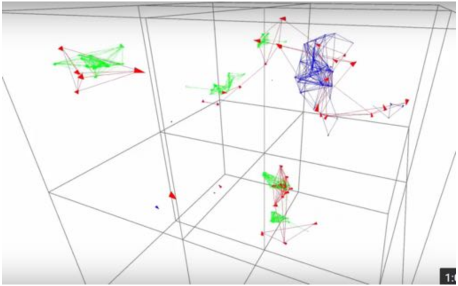
$$\begin{array}{l}\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i), \mathbf{u}_i \in [\mathbf{u}_i].\\ g\left(\mathbf{x}_{i(k)}(a(k)), \mathbf{x}_{j(k)}(b(k)), a(k), b(k)\right) = 0\\ \tilde{a}(k) = h_{i(k)}(a(k))\\ \tilde{b}(k) = h_{j(k)}(b(k))\\ \dot{h}_i = 1 + n_h,\ n_h \in [n_h]\end{array}$$



[Youtube [1]]



Youtube



[Youtube]

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